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BUCKLING OF A ROD UNDERGOING DIRECT OR REVERSE MARTENSITE TRANSFORMATION UNDER COMPRESSIVE STRESSES

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Analytical solutions of the problem of buckling of a compressed rod made of a shape-memory alloy, which undergoes direct or reverse martensite phase transition under compressive stresses, are obtained with the use of various hypotheses. Specific features of the experimentally observed buckling phenomenon caused by martensite transformations are described. It is found that the hypotheses of continuing phase transition and continuing loading give the minimum critical loads.

Key words: *buckling of a rod, critical loads, martensite transformations.*

Buckling of thin-walled elements made of shape-memory alloys (SMA) was considered in few papers [1–7]. Movchan et al. [5, 6] found experimentally that thermoelastic martensite phase transitions responsible for unique mechanical properties of SMAs can lead to buckling. The critical loads in the direct transformation were found to be smaller than those for isothermal buckling in the least severe martensite phase state. The critical loads in the reverse transformation from the martensite state, which is free of phase strains responsible for shape distortion, lie between the limit forces mentioned above. For a fixed load, the buckling temperature increases with the length of the working part of a specimen in the direct transformation and decreases in the reverse transformation.

The present paper addresses the stability problem of a rod made of an SMA that undergoes direct and reverse martensite transformations under the action of compressive stresses. Hypotheses that can be used to solve the problem are analyzed.

1. Formulation of the Problem. We consider an initially straight rod made of an SMA with a constant cross section, symmetric about the principal axes of inertia. The lower end of the rod is hinged and the upper end is simply supported. A dead compressive force P is applied to the upper end of the rod, which produces an axial stress σ distributed uniformly over the cross section. In the case of the direct martensite transformation, the buckling analysis is performed under the assumption that the force is applied in the austenite state at a temperature so high that the stresses produced by this force cannot cause the direct martensite transformation under isothermal conditions. Then, the rod is left to cool from the initial to final temperature of the direct martensite transformation. In the case of the reverse martensite transformation, the buckling analysis is performed under the assumption that the rod in the martensite state is loaded by the force P producing a stress σ insufficient for SMA deformation by twinning. The rod has an initial compressive phase strain $\varepsilon^{(0)}$ and is slowly heated from the initial to final temperature of the reverse transformation. The strain $\varepsilon^{(0)}$ can be produced in the rod by preliminary cooling from the initial to final temperature of the direct martensite transformation under the action of a fixed compressive load R that produces a stress σ_0. It is assumed that, in this case, no buckling occurs at the stage of the preliminary direct transformation and, hence, the stresses σ_0 are uniformly distributed over the cross section.

In the prebuckling state, all points of the rod have the same temperature. It is required to find the loads (rod lengths) for which curvilinear equilibrium configurations exist in the case of the direct or inverse transformation in addition to the trivial (rectilinear) configuration of the rod.

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The analysis is based on the theory of small strains and the hypothesis of plane cross sections (for total strains). The buckling problem is solved in a linearized formulation. Since the rod is uniformly compressed before its transition to the contiguous state of equilibrium, the total, elastic, and phase strains and stresses in the prebuckling state are identical at each point of the rod, the prebuckling curvature and deflection being equal to zero. The prebuckling temperature of all points of the rod is assumed to be identical at each time.

In the buckling analysis, it is necessary to infer which quantities can be varied to obtain equations for the perturbed state. We assume that, for slow variation in temperature, an additional phase transition has no time to occur in the case of the fast transition to the contiguous state of equilibrium. Therefore, in the linearized buckling equations, we ignore variation in the phase-composition parameter. For brevity, we call this approach the hypothesis of a fixed phase composition [7]. Strictly speaking, this concept is valid for SMAs of the CuMn type for which the phase composition is determined by temperature only and does not depend (or depends only slightly) on acting stresses.

At the same time, for most SMAs, the thermoelastic martensite transition can be caused not only by cooling or heating but also by variation in stresses. Since the stresses vary in the transition to the contiguous state of equilibrium, the phase composition also varies; therefore, the phase-composition parameter should be varied. For brevity, we call this hypothesis the concept of a continuing phase transition [7].

We assume that the load does not vary during the transition to the contiguous state of equilibrium. In this case, the perturbed state is characterized by an unloading zone formed near the concave surface of the rod, in which the additional phase transition ceases because of stress reduction. For brevity, we call this hypothesis the concept of elastic unloading. If we assume that the acting load is varied, there may exist infinitesimal variations in the load such that the unloading zone vanishes. As a result, the entire cross section is in the region of the additional phase transition. This hypothesis corresponds to Shenley's concept in the stability theory for elastoplastic bodies and, hence, it can be called the concept of continuing loading [8, 9].

Temperature can also be considered as either a varied or unvaried parameter. In the first case, it is natural to assume that, in contrast to temperature, its variation depends on the coordinates of the cross-sectional points. If the temperature is varied, even for SMAs whose phase composition is independent of acting stresses, the variation in the phase-composition parameter is also nonvanishing; moreover, this variation can, generally, be different at different points of the cross section.

Movchan et al. [5, 6] found experimentally that the critical loads are very low. Therefore, we use hypotheses that yield the minimum critical force or minimum critical length (for a fixed load). This condition is equivalent to the condition of the minimum absolute value of the coefficient of curvature variation in the expression for the variation in the bending moment.

2. System of Constitutive Relations. To solve the stability problem, we use a simplified version of the system of constitutive relations for the SMA, which were proposed in [10–13]. In the one-dimensional case (for the axial stresses σ and strains ε), this system becomes

$$\varepsilon = \varepsilon^{(1)} + \varepsilon^{(2)}; \tag{2.1}$$

$$d\varepsilon^{(2)} = (2c_0\sigma/3 + a_0\varepsilon^{(2)})\, dq; \tag{2.2}$$

$$d\varepsilon^{(2)} = (a_0\varepsilon^{(0)}/(\exp{(a_0)} - 1) + a_0\varepsilon^{(2)})\, dq; \tag{2.3}$$

$$q = \sin\Big(\frac{\pi}{2}\,\frac{M_{\rm in} + k|\sigma| - T}{M_{\rm in} - M_{\rm fin}}\Big), \qquad q = \sin\Big(\frac{\pi}{2}\,\frac{A_{\rm fin} + k|\sigma| - T}{A_{\rm fin} - A_{\rm in}}\Big); \tag{2.4}$$

$$q = \frac{1}{2}\Big[\cos\Big(\pi\,\frac{T - M_{\rm fin} - k|\sigma|}{M_{\rm in} - M_{\rm fin}}\Big) + 1\Big], \qquad q = \frac{1}{2}\Big[\cos\Big(\pi\,\frac{T - A_{\rm in} - k|\sigma|}{A_{\rm fin} - A_{\rm in}}\Big) + 1\Big]; \tag{2.5}$$

$$\varepsilon^{(1)} = \sigma/E(q); \tag{2.6}$$

$$1/E(q) = q/E_1 + (1-q)/E_2. \tag{2.7}$$

Here a_0, c_0, and k are the material parameters, ε, $\varepsilon^{(1)}$, and $\varepsilon^{(2)}$ are the total, elastic, and phase strains, respectively, q is the volume fraction of the martensite phase, which can be determined by either relations (2.4) [13] or (2.5) [14], T is the temperature of the rod cross section, and E_1 and E_2 are Young's moduli for martensite and austenite states, respectively. The first equations in (2.4) or (2.5) and relation (2.2) refer to the direct transformation, whereas the second relations in (2.4) or (2.5) and relation (2.3) refer to the reverse transformation; M_{in}, M_{fin} and A_{in}, A_{fin} are the initial and final temperatures of the direct and reverse martensite transformations in the stress-free material, respectively. For the reverse transformation from a completely martensite state, the quantity $\varepsilon^{(0)}$ in expression (2.3) is the phase strain at the point of the beginning of the reverse transformation. Expression (2.7) for Young's modulus E is a result of the assumption of additivity of the Gibbs potential and averaging according to Reuss [15].

Relations (2.4) and (2.5) do not take into account the dependence of the transition temperature on shear stresses that occur due to bending. According to Eqs. (2.1)–(2.3), we ignore the volume effect of the martensite-transformation reaction and temperature strain. In the case of an arbitrarily specified preliminary phase strain, relation (2.3) ignores the reverse shape-memory effect [13]. If the preliminary phase strain $\varepsilon^{(0)}$ in the reverse transformation occurs as a result of the direct transformation under the action of a constant stress σ_0, the constitutive relation (2.3) takes this effect into account, and integration of relation (2.2) for $\sigma = \sigma_0 = \text{const}$ and zero initial conditions yields $\varepsilon^{(0)} = 2c_0\sigma_0(\exp(a_0) - 1)/(3a_0)$, i.e., in this case, relation (2.3) can be written as

$$d\varepsilon^{(2)} = (2c_0\sigma_0/3 + a_0\varepsilon^{(2)})\,dq. \tag{2.8}$$

Consequently, the constitutive relations for direct (2.2) and reverse (2.8) transformations differ only in that the direct-transformation equation contains the stress that acts during the direct transformation, whereas the reverse-transformation equation contains a constant stress that acted during the preliminary direct transformation.

For the prebuckling uniform strain occurring in the direct transformation or reverse transition, the dependence $\varepsilon^{(2)}(q)$ can be found by integrating the constitutive relation (2.2) or (2.8) under zero initial conditions. As a result, for the reverse transformation, we obtain

$$\varepsilon^{(2)} = 2c_0\sigma_0[\exp(a_0 q) - 1]/a_0. \tag{2.9}$$

For the direct transformation, the quantity σ_0 in formula (2.9) should be replaced by σ.

3. Solution for all Parameters Varied. For the total strain, the hypothesis of plane cross sections is written as

$$\varepsilon = \varepsilon_0 + \beta z = \sigma/E(q) + \varepsilon^{(2)}. \tag{3.1}$$

Here z is the transverse coordinate in the plane of bending, ε_0 is the strain of the neutral line,

$$\beta = \frac{\partial^2 w}{\partial x^2} \tag{3.2}$$

is the curvature, w is the deflection, and x is the axial coordinate. In this section, the entire set of variables is varied (q, P, and T), i.e., the analysis is performed with the use of the hypotheses of a continuing phase transition and continuing loading with a varied temperature.

Varying both sides of equality (3.1), for the reverse transformation, we obtain

$$\delta\varepsilon_0 + z\,\delta\beta = A\,\delta\sigma + (B\sigma + C\sigma_0)\,\delta q,$$
$$A = q/E_1 + (1-q)/E_2, \qquad B = 1/E_1 - 1/E_2, \qquad C = 2c_0\exp(a_0 q)/3. \tag{3.3}$$

These relations are obtained with allowance for the varied relations (2.6) and (2.7) and relation (2.8), in which the differential sign is replaced by the variation sign and the phase strain $\varepsilon^{(2)}$ is replaced by expression (2.9).

Varying the second formulas in (2.4) or (2.5), for the reverse transformation, we obtain

$$\delta q = \gamma f(q)(k\,\delta|\sigma| - \delta T)H(\delta T - k\,\delta|\sigma|), \qquad \gamma = 1/(A_{\text{fin}} - A_{\text{in}}), \tag{3.4}$$

where $f(q) = (\pi/2)\sqrt{1-q^2}$ for relation (2.4), $f(q) = \pi\sqrt{q(1-q)}$ for relation (2.5), and $H(x)$ is the Heaviside function. For the direct transformation, we obtain a relation similar to (3.4), in which $\gamma = 1/(M_{\text{in}} - M_{\text{fin}})$ and the argument of the Heaviside function is the quantity $k\,\delta|\sigma| - \delta T$.

Substitution of (3.4) into (3.3) yields

$$\delta\varepsilon_0 + z\,\delta\beta = A\,\delta\sigma + \gamma(B\sigma + C\sigma_0)(k\,\delta|\sigma| - \delta T)f(q)H(\delta T - k\,\delta|\sigma|). \tag{3.5}$$

Since fixed compressive stresses $\sigma_0 < 0$ and $\sigma < 0$ act during the direct and reverse transformations, both stresses satisfy the equalities $|\sigma| = -\sigma$ and $\delta|\sigma| = -\delta\sigma$. Introducing the notation $S_0 = -P_0/F = -\sigma_0$ and

$S = P/F = -\sigma$ or $S = |\sigma|$ and $\delta|\sigma| = \delta S$ (or $\delta\sigma = -\delta S$) in (3.5) and expressing the variation of S from the resulting relation, we obtain

$$\delta S = \begin{cases} -(\delta\varepsilon_0 + z\,\delta\beta - B_2\,\delta t)/(A + B_2), & \delta\varepsilon_0 + z\,\delta\beta + A\,\delta t > 0, \\ -(\delta\varepsilon_0 + z\,\delta\beta)/A, & \delta\varepsilon_0 + z\,\delta\beta + A\,\delta t < 0. \end{cases} \tag{3.6}$$

Here $\delta t = \delta T/k$ and $B_2 = k(BS + CS_0)\gamma f(q)$. Since the term B_2 is nonnegative and the variation δt is arbitrary, the minimum absolute value of the coefficient of the curvature variation in the expression for the bending moment variation corresponds to the case where condition (3.6) holds for the entire cross section, i.e., an additional phase transformation occurs in the entire cross section upon the transition to the contiguous state of equilibrium.

It should be noted that, for any infinitesimal variations $\delta\beta$ and δt, there always exists an infinitesimal variation δP such that condition (3.6) holds for the entire cross section of the rod. This value of δP can also be found in the absence of temperature variation. Indeed, evaluating the bending moment variation with allowance for the first formula (3.6), expressing it in terms of deflection variation with the use of (3.2), and writing the equation of equilibrium for the moments, we obtain

$$\frac{\partial^2 \delta w}{\partial x^2} + \frac{(A + B_2)P}{J}\,\delta w - \frac{B_2}{J}\int\limits_{\text{fin}} z\,\delta t_1\,dz = 0. \tag{3.7}$$

Here δt_1 is the antisymmetric part of the variation δt and J is the moment of inertia. By its form, relation (3.7) corresponds to the equation of elastic stability of the same rod in the presence of a small transverse load, which is known to have no effect on the critical force. Thus, the symmetric part of the temperature variation does not affect the stability equation, whereas taking into account the antisymmetric part leads to an inhomogeneous equation, which, however, has no effect on the critical force. Consequently, whether or not the temperature is varied (this does not affect the critical force) is of no significance if the hypotheses of a continuing phase transition and continuing loading are used.

As in the case of the elastic stability analysis, Eq. (3.7) yields the following relation between the force P, rod length L, and parameter q, which corresponds to the first buckling mode:

$$L = \pi\sqrt{\frac{E(q)J}{(1 + kE(q)(BS + CS_0)\gamma f(q))P}}\,. \tag{3.8}$$

Using (3.8), the dimensionless length of the rod $\lambda = L/L_1$ in the reverse transformation can be expressed in terms of the dimensionless load parameters [this parameters refer to the stages of the direct transformation ($r = k\gamma R/F$) and reverse transformation ($p = k\gamma P/F$)] and dimensionless material parameters $e = E_1/E_2$ and $c = c_0 E_1$:

$$\lambda^{-2}(p, r, q) = q + e(1 - q) + (p(1 - e) + (2/3)rc\exp(a_0 q))f(q). \tag{3.9}$$

Here $L_1 = \pi\sqrt{E_1 J/P}$ is the critical length of the rod for isothermal buckling in the martensite state. To describe buckling in the direct transformation, it is necessary to set $r = p$ in formula (3.9) and calculate the load parameter using the value of γ characteristic of the direct transformation.

The load parameter p has a clear physical meaning. The quantity kP/F is equal to the increase in the transition temperature caused by prebuckling stresses. Therefore, the parameter p is equal to the ratio of the "force" shift of the transition temperatures to the difference between the final and initial transformation temperatures. It is worth noting that the dimensionless parameter of the load r that acts at the preliminary stage of the direct transformation is calculated for the value of γ characteristic of the reverse transformation.

4. Solution for the Hypothesis of Elastic Unloading. Let the axial force and temperature remain unvaried upon the transition to the contiguous state of equilibrium:

$$\int\limits_{\text{fin}} \delta S\,dF = \delta P = 0, \qquad \delta T = 0. \tag{4.1}$$

For certainty, we consider the buckling for the reverse transformation. Under these conditions, the variation in the quantity S becomes

$$\delta S = \begin{cases} -(\delta\varepsilon_0 + z\,\delta\beta)/(A + B_2), & \delta\varepsilon_0 + z\,\delta\beta \geqslant 0, \\ -(\delta\varepsilon_0 + z\,\delta\beta)/A, & \delta\varepsilon_0 + z\,\delta\beta \leqslant 0. \end{cases} \tag{4.2}$$

For simplicity, we consider the case of a rectangular cross section of height h. Using relation (4.1), we calculate the coordinate z_0 of the cross-sectional point at which the nominator in (4.2) changes the sign:

$$\zeta_0 = -(1+2\xi) + 2\sqrt{\xi(1+\xi)}, \qquad \zeta_0 = 2z_0/h, \qquad \xi = A/B_2. \tag{4.3}$$

Multiplying (4.2) by z and integrating over the cross-sectional area with allowance for (4.3) and (3.2), we obtain the variation in the bending moment

$$\delta M = -\frac{bh^3\zeta_0}{3B_2}\frac{\partial^2 \delta w}{\partial x^2}. \tag{4.4}$$

Using the standard technique for solving stability problems and taking into account (4.4), from the equations of equilibrium for the moments, we obtain

$$4\lambda^{-2} = 2\varphi(q) + \psi(q) + 2\sqrt{\varphi(q)(\varphi(q)+\psi(q))}\,; \tag{4.5}$$

$$\varphi(q) = q + e(1-q), \qquad \psi(q) = ((1-e)p + (2/3)cr\exp(a_0 q))f(q). \tag{4.6}$$

For the direct transformation, the stability problem is solved with the use of formulas (4.5) and (4.6) for $r = p$.

5. Analysis of Results. Setting $k = 0$ in (3.9) or (4.5) and (4.6) (which is equivalent to $p = 0$ and $r = 0$), we obtain the standard Euler formula with a variable Young's modulus. This solution corresponds to the assumption of the absence of the additional phase transformation upon the transition to the contiguous state of equilibrium: $\delta q = 0$, i.e., the hypothesis of a fixed phase composition [4]. By virtue of positiveness of the second term in the denominator in (3.8) or the function $\psi(q)$ in (4.6), the critical length calculated with allowance for the variation in the phase-composition parameter is always smaller than that obtained without allowance for this variation for the same value of Young's modulus.

Whether or not the hypothesis of continuing loading or elastic unloading is accepted, the solution based on the hypothesis of continuing phase loading and approximations (2.4) and (2.5) shows that the quantity λ decreases in the neighborhood of the point $q = 0$ and increases in the neighborhood of the point $q = 1$ with an increase in q. It is found that the curves $\lambda = \lambda(q)$ obtained with the use of the hypotheses of a continuing phase transition [7] and continuing loading [8, 9] lie below the corresponding curves obtained for other hypotheses.

At a certain intermediate point $q^* \in (0, 1)$, the quantity λ reaches the unique minimum λ^*. As $q \to 1$, we obtain $f(q) \to 0$ for approximations (2.4) and (2.5), and hence, $\lambda \to 1$, i.e., the critical length of the rod increases and tends to the critical length of isothermal buckling in the martensite state L_1. It follows that the minimum critical length of buckling caused by the direct or reverse martensite transformation, determined with the use of the concept of a continuing phase transition, is always smaller than the critical length of isothermal buckling in the martensite state. This inference agrees with the experimental data of [5, 6].

For approximation (2.5) of the phase diagram, the critical length of the rod tends to the critical length $L_2 = \pi\sqrt{E_2 J/P}$ as $q \to 0$, which corresponds to isothermal buckling in the austenite state. Consequently, for approximation (2.5), the dependence of the critical length of buckling caused by the thermoelastic martensite transformation on q is represented by a continuous curve connecting the point $q = 0$, $L = L_2$ (elastic buckling in the austenite state) and the point $q = 1$, $L = L_1$ (elastic buckling in the martensite state). This curve lies below the curve of isothermal buckling that corresponds to Eq. (3.9) or (4.5) and (4.6) for $k = 0$ ($p = 0$ and $r = 0$).

If approximation (2.4) is used for the transition diagram, it follows from (3.9) or (4.5), (4.6) that the critical length of the rod tends to a certain value smaller than L_2 as $q \to 0$. Hence, in this case, the dependence of the critical length on the phase-composition parameter has a discontinuity at the point of the beginning of the direct martensite transformation. In the interval $q \in [0, 1/3]$, the critical length of the rod calculated with the use of approximation (2.4) is smaller than that obtained for approximation (2.5). For $q = 1/3$, both approximations give the same results; for $1/3 < q < 1$, the critical force calculated for approximation (2.5) is smaller than that for (2.4).

Figure 1 shows the dimensionless critical length of the rod λ as a function of q, calculated for the direct transformation for $p = 1$ by formulas (3.9) (curves 1) and (4.5) and (4.6) (curves 2). The solid and dashed curves refer to approximations (2.5) and (2.4) of the transition diagram, respectively. Curve 3 refers to the solution obtained for the hypothesis of a fixed phase composition. Here and below, the calculations are performed for the dimensionless parameters $e = 1/3$ and $c = 7.9$ characteristic of titanium nickelide [13].

Since the critical length of the rod is minimum for $q > 1/3$ for both approximations of the phase diagram, approximation (2.5) gives a smaller value of the minimum critical force, compared to approximation (2.4). Figure 2 shows the minimum dimensionless critical length of buckling λ versus the dimensionless load parameter p for the

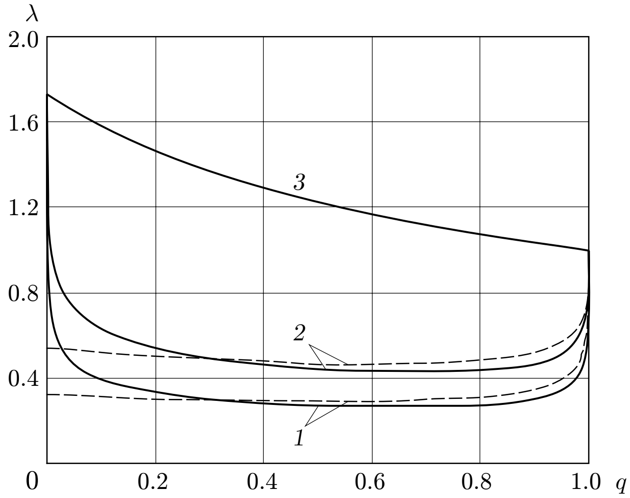

Fig. 1

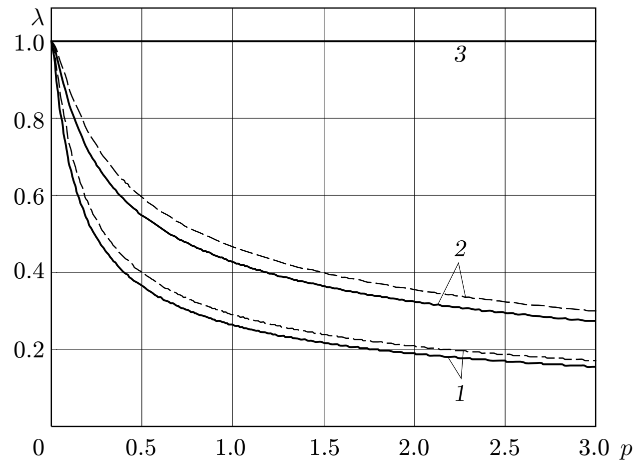

Fig. 2

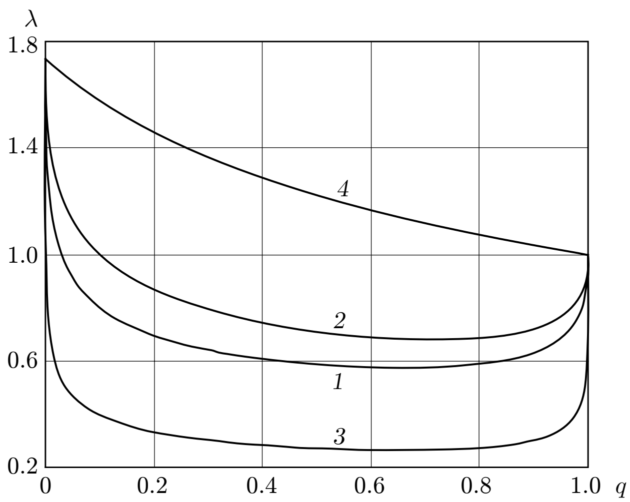

Fig. 3

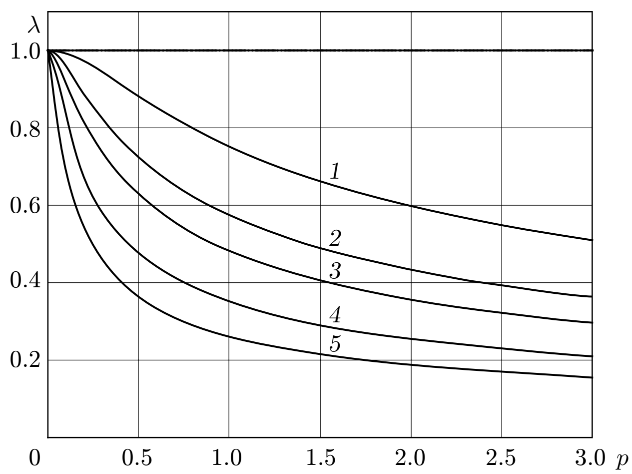

Fig. 4

direct martensite transformation (notations the same as in Fig. 1). One can see from Fig. 2 that, for reasonably high loads, the critical length of buckling caused by the direct martensite transformation can be much smaller than that of the isothermal Euler buckling for the minimum (martensite) value of Young's modulus. This is in agreement with the experimental data of [5, 6]. The minimum critical length determined for the hypothesis of continuing loading is always smaller than that determined for the hypothesis of elastic unloading.

Let $M_{\rm in} - M_{\rm fin} = A_{\rm fin} - A_{\rm in}$, i.e., the values of the parameter γ for the direct and reverse transformations are identical. One can show that, in this case, the critical length of buckling in the reverse transformation with the loads p and r always lies between the critical lengths of buckling in the direct transformation for the loads p and r. In Fig. 3, curve 1 refers to buckling in the reverse transformation under the load $p = 1$ after the direct transformation under the load $r = 0.1$. Curves 2 and 3 refer to buckling in the direct transformation under the loads $p = 0.1$ and 1.0. Curve 4 was obtained with the use of the hypothesis of a fixed phase composition.

For a specified load acting in the reverse transformation, the critical length of the rod is a decreasing function of the compressive load that acts at the preliminary stage of the direct transformation. Figure 4 shows the minimum dimensionless critical length of rod buckling in the reverse transformation versus the dimensionless load p for various values of r. The solution was obtained for the hypothesis of continuing loading. Curve 1 refers to $r = 0$ and curves 2–5 refer to $r = 0.1p$, $0.2p$, $0.5p$, and p, respectively. Curve 1 refers to buckling caused by the reverse transformation from the martensite state in the absence of phase strains responsible for shape distortion. These critical lengths are much smaller than the minimum critical length of isothermal buckling shown by the dashed line

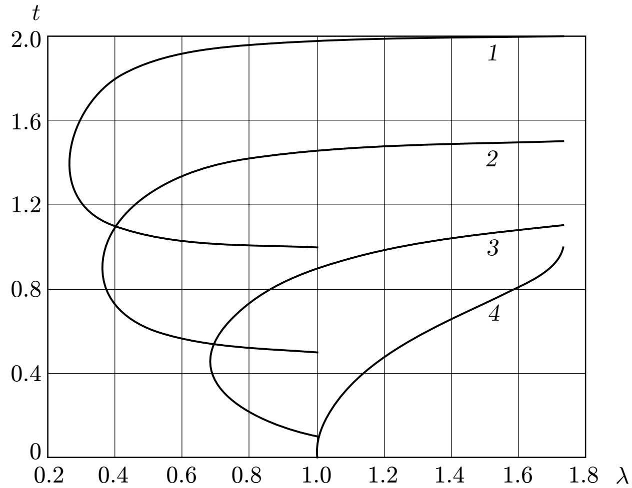

Fig. 5

$\lambda = 1$. At the same time, they exceed the critical length of buckling caused by the direct transformation under the same load (curve 5). These findings also agree with the experimental data of [5, 6].

Relation (2.5) implies the equality

$$t = p + \arccos(2q - 1)/\pi, \tag{5.1}$$

where $t = (T - M_{\rm fin})/(M_{\rm in} - M_{\rm fin})$ for the direct transformation and $t = (T - A_{\rm in})/(A_{\rm fin} - A_{\rm in})$ for the reverse transformation. Figure 5 shows the dimensionless buckling temperature t versus λ, calculated by (5.1) and (3.9). The portions of the curves above the points at which the curves have vertical tangents refer to the direct transformation and the portions of the curves below these points refer to the reverse transformation for $p = r$. Curves 1–4 refer to $p = 1$, 0.5, 0.1, and 0 ($k = 0$), respectively. One can see from Fig. 5 that as, the specimen length increases, the buckling temperature increases for the direct transformation and decreases for the reverse transformation (except for curve 4 obtained for the hypothesis of a fixed phase composition), which agrees with the experimental data of [5, 6].

Thus, the linearized stability analysis of a rod made of an SMA, which was performed under the hypothesis of a continuing phase transformation and continuing loading, gives a qualitative description of the experimentally observed buckling [5, 6] caused by martensite phase transformations under the action of compressive stresses.

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